Weak Discriminative Verification Enables Strong Test-time Scaling

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Abstract

Test-time scaling has become a popular strategy to boost large language model performance on complex reasoning tasks. A standard approach involves sampling multiple candidate solutions, then selecting the final answer via self-consistency or a verifier model. While generative verifiers can outperform self-consistency, they incur substantial compute overhead due to expensive chain-of-thought generation, and often yield limited gains under practical budget constraints. Discriminative verifiers, by contrast, are far more efficient but typically underperform self-consistency on challenging reasoning tasks when the pool of candidate solutions grows large.

In this work, we show that a weak discriminative verifier can be transformed into a strong test-time scaler, achieving both efficient verification and strong downstream performance. Specifically, by pairing a lightweight discriminative verifier with a simple pessimism penalty that down-weights low-support answers, our method can consistently outperform self-consistency with minimum overhead in verification compute. On AIME2024, DeepSeek-R1-Distill-Qwen-32B paired with our method improves from 68.2% to 79.7% with just 4 candidate solutions – matching the performance of o3-mini (medium) and outperforming self-consistency by 2.2% for only 0.5% additional compute. Our results suggest that lightweight discriminative verification with pessimistic scoring offers a practical and efficient solution to test-time scaling. Code is available at `https://anonymous.4open.science/r/DPV-NeurIPS2025`.

1 Introduction

Since the release of OpenAI's o1 OpenAI (2024), there has been substantial progress in enhancing the reasoning capabilities of large language models (LLMs) by scaling test-time compute (Snell et al., 2024) across domains such as mathematics, coding, and general problem solving. Broadly speaking, test-time scaling refers to strategies that can improve model performance on downstream tasks by allocating additional computational resources during inference. A canonical example is self-consistency (SC) (Wang et al., 2023b), which involves sampling multiple completions from the model and selecting the final answer via a majority vote. Alternatively, one can enhance answer selection by employing a verifier model that scores and ranks candidate solutions based on their likelihood of correctness.

Initial approaches to verification relied on discriminative models that output scalar correctness scores for individual solutions or steps (Cobbe et al., 2021; Lightman et al., 2023). More recently, some works have explored leveraging the generative abilities of LLMs for verification purposes. These

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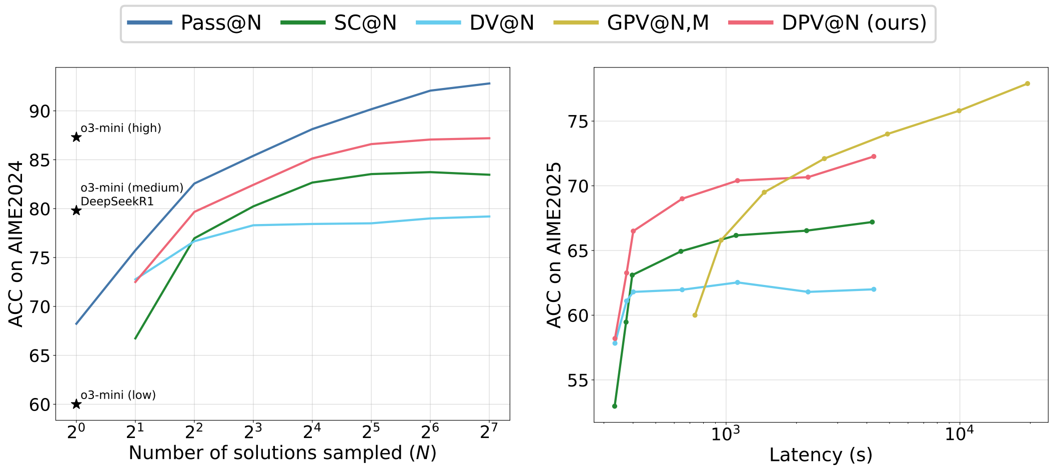


Figure 1: **Left:** Accuracy rates of DeepSeek-R1-Distill-Qwen-32B on AIME2024 under various selection algorithms. DV underperforms SC for all but small values of N, but DPV maintains a sizable margin over SC as N scales. DPV matches the accuracy of DeepSeek-R1 and o3-mini (medium) when $N = 4$ and ties o3-mini (high) with additional compute. **Right:** DPV consistently outperforms SC for a negligible amount of additional compute on AIME2025, and even outperforms GPV (Shi & Jin, 2025) for most practical inference budgets. N is doubled at each point along the x-axis. For GPV, each solution is verified twice.

models, also known as generative verifiers (Zhang et al., 2024c; Mahan et al., 2024), produce chain-of-thought (CoT) rationales before outputting a verdict. This generative approach opens up a new avenue for test-time scaling: increasing the number of verification passes over candidate solutions. As a result, there is a growing number of works studying how to achieve stronger test-time scaling through scaling verification compute (Zhao et al., 2025; Shi & Jin, 2025)

While generative verifiers generally offer stronger performance, they require significantly more compute to do so. These verifiers are often reasoning-heavy models that generate lengthy CoTs before producing a verdict, resulting in overhead that rivals or even exceeds the cost of generating the candidate solutions. Indeed, recent work by Singhi et al. (2025) demonstrates that when verification cost is properly accounted for, generative verifiers underperform SC under low inference budgets. In fact, they require up to $8\times$ more compute just to match SC, and deliver marginal gains (3.8%) even when granted $128\times$ the compute budget.

Why is scaling verification-time compute often less effective than scaling compute for solution generation? A key reason is that solution correctness is ultimately bottlenecked by the quality of the candidate solutions sampled from the solver. If the solver fails to produce any correct candidates, no verifier—regardless of strength—can recover the correct answer. Moreover, the SC baseline is already quite strong, nearing pass@N on many tasks. To surpass SC, a verifier must both (1) agree with the majority when it is correct, and (2) successfully identify the correct minority solution when the majority is wrong. These requirements make it difficult for a verifier to deliver significant gains, especially under a fixed compute budget. As a result, allocating additional compute to generating candidate solutions typically yields better returns than spending it on verification.

Given these limitations, it is preferable to minimize the cost of verification under constrained budgets. Discriminative verifiers present a promising alternative due to their computational efficiency. Unlike generative verifiers, which require both a costly prefilling step and sequential token generation during decoding, discriminative verifiers only perform a single forward pass (i.e., prefilling), avoiding the decoding bottleneck. However, despite their speed advantage, discriminative verifiers exhibit limited capabilities on complex reasoning tasks (Tan et al., 2025), often underperforming SC as the pool of candidate solutions grows.

In this work, we show that a weak discriminative verifier can nevertheless be transformed into an effective test-time scaler, offering the best of both worlds: consistent improvements over SC with minimal verification overhead. The key insight is to exploit the signal from the already strong

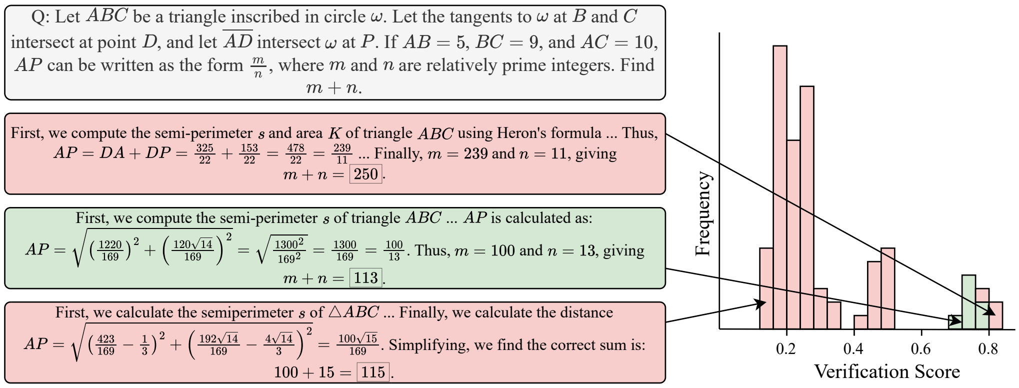


Figure 2: An example question and candidate solutions from AIME2024, along with the distribution of verification scores. The correct answer is 113. SC simply selects the most frequent answer, which in this case is 115, while DV is misled by a high-scoring distractor and selects the answer 250. Meanwhile, by discounting the verification scores by each answer's support among the candidates, DPV identifies the correct answer.

SC baseline and enhance it with additional, lightweight verification scores from the weak verifier. Inspired by the pessimistic verification strategy of Shi & Jin (2025), which penalizes overconfident reward estimates in generative models, we propose a discriminative variant adapted to our setting. This method allows even a small 7B verifier to act as an effective test-time scaling module. As Figure 1 demonstrates, while this weak verifier underperforms SC when used in isolation (e.g., for Best-of-N selection), combining it with our pessimistic scoring mechanism consistently improves over SC on AIME2024 (as well as other math and general reasoning benchmarks), all while keeping verification costs a tiny fraction over the generation compute (<1.5%). With our test-time scaling method, we can improve the AIME2024 accuracy of DeepSeek-R1-Distill-Qwen-32B from 68.2% to 79.7% with only 4 candidate solutions, matching the performance of o3-mini (medium) and outperforming SC by 2.2%.

Our contributions are as follows:

- We introduce a novel *discriminative pessimistic verification* (DPV) strategy, which discounts the verifier's score by an answer's support among the candidate solutions, to enhance verifier reliability.
- We empirically validate our approach across various reasoning tasks, showing that it consistently outperforms SC and DV methods across a number of settings for a negligible amount of additional compute.
- We demonstrate that our approach outperforms generative verification under practical compute budgets, enabling strong test-time scaling through weak verification.

2 Preliminaries

Repeated sampling. Repeated sampling is a test-time scaling technique that involves generating a batch of N independent candidate solutions and then selecting a final answer from among them. As N increases, the probability that at least one solution is correct also rises (i.e., Pass@N improves; see Figure 1) (Cobbe et al., 2021). However, this leaves open the central challenge of selecting a final answer from among the candidates in the absence of ground truth.

Self-consistency. Self-consistency (SC) (Wang et al., 2023b) addresses this by grouping responses by their final answer and taking the majority vote. While this approach is robust when the correct answer is common, it can fail when a compelling but incorrect answer is dominant among the candidates.

Best-of-N. Another strategy is best-of-N (BoN) selection (Charniak & Johnson, 2005; Cobbe et al., 2021), which uses a *verifier* to score each solution and selects the highest-scoring one. A strong verifier can identify correct but rare responses that SC might miss. However, as N increases, it can also be misled by confident yet incorrect responses, highlighting a long-tail vulnerability. Verifiers come in two forms:

- *Discriminative verifiers* (or reward models) (Cobbe et al., 2021) assign a scalar score (e.g., in $[0, 1]$) to each response. We refer to this setting as *discriminative verification* (DV).
- *Generative verifiers* (Zhang et al., 2025) prompt an LLM to judge correctness via free-form CoT reasoning. Generative verifiers can benefit from inference-time scaling by independently sampling multiple verification chains and aggregating their outputs for a more robust verdict.

Pessimistic Best-of-N. To guard against the long-tail of high-scoring but incorrect responses, one can subtract a *pessimism* penalty from each verifier score, analogous to a lower-confidence bound in multi-armed bandits (Auer et al., 2002). For example, Shi & Jin (2025) penalizes a generative verifiers scores proportional to $\ln(NM)/(n_k M + 1)$ for each answer cluster of size n_k, effectively interpolating between best-of-N and self-consistency. When paired with a discriminative verifier, we refer to it as *discriminative pessimistic verification* (DPV).

3 Effective Discriminative Verification

In this section, we outline the techniques that we use to train our discriminative verifier, as well as the discriminative pessimistic verification (DPV) algorithm that we use to perform effective test-time scaling.

3.1 Training a discriminative verifier

Dataset curation. We sample 32k math problems from NuminaMath (LI et al., 2024), which aggregates problems from Chinese K-12 exams, Orca-Math (Mitra et al., 2024), AoPS forums, and various Olympiads (e.g., IMO, APMO, BMO), among other sources. We decontaminate the training dataset by excluding any problem whose fuzzy-match similarity to an entry in our evaluation sets exceeds 80. For each question, we sample one response from each of ten LLMs: DeepSeek-R1 and its six distilled variants (DeepSeek-AI et al., 2025), DeepScaleR-1.5B-Preview (Luo et al., 2025b), and both the preview and production releases of QWQ-32B (Team, 2024, 2025). We grade each response for correctness using HuggingFace's Math-Verify toolkit (Kydlíček, 2025), which parses the model's final answer and performs symbolic equivalence checks against the reference solution. We throw out problems for which all ten solutions are either correct or incorrect, leaving just 11,670 response groups for training.

Verifier training. Following prior work (Qwen et al., 2025; Yang et al., 2024), we replace the language modeling head of the LLM (specifically DeepSeek-R1-Distill-Qwen-7B) with a two-layer scaler value head. We train our verifier using a Bradley-Terry ranking loss combined with an L_2 regularization term (Ouyang et al., 2022; Kirchner et al., 2024). Concretely, our loss is

$$\mathcal{L} = -\frac{1}{|P|\,|N|} \sum_{i \in P} \sum_{j \in N} \log \sigma\big(r_i - r_j\big) \; + \; \frac{\lambda}{2}\, \mathbb{E}\left(r^2\right),$$

where $r = (r_1, \ldots, r_m)$ are the logits assigned by the verifier to a batch of m responses, $\sigma(x)$ is the logistic function, and P and N are the sets of correct and incorrect responses, respectively. The first term implements the Bradley–Terry model by maximizing the probability $\sigma(r_i - r_j)$ that every correct response $i \in P$ outranks every incorrect response $j \in N$ (Bradley & Terry, 1952), and the second term keeps score head well-behaved and centered around zero. By computing all $|P| \times |N|$ comparisons in one vectorized pass instead of sampling pairs, we gain both higher throughput and more stable gradients. Additional training details, including hyperparameters are provided in Appendix A.

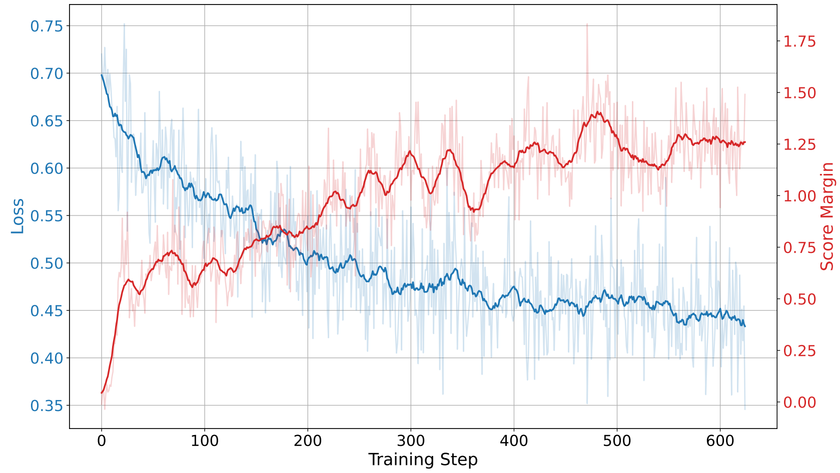


Figure 3: **Blue:** The loss decreases over one epoch of training. **Red:** The score margin—the difference in score assigned to correct solutions and incorrect solutions on average across a global batch—increases during training. Together, these indicate the discriminative verifier learns to discriminative between correct and incorrect solutions.

3.2 Discriminative pessimistic verification

Repeatedly sampling independent solutions boosts the chance that at least one is correct (improving pass@N), but leaves open the question of which answer to select. SC (Wang et al., 2023b) tends to ignore rare yet correct solutions, while DV Cobbe et al. (2021) can catch rare solutions but is prone to selecting high-scoring distractors in the long tail. In practice, reliably selecting the final answer requires balancing the verifier's confidence in each candidate solution against its support among the other candidates.

To address this, Shi & Jin (2025) introduces *pessimistic verification* in which the N responses are grouped by their final answer a and assigned a penalized score of the form

$$\mathrm{Score}(a_k) \;=\; \bar{r}(a_k) \;-\; \alpha\,\Psi(N,\,n_k),$$

where n_k is the support or frequency of a_k, $\bar{r}(a_k)$ is the mean score over n_k verifications, and $\Psi\colon \mathbb{N}\times\mathbb{N}\to\mathbb{R}_{\geq 0}$ is any non-increasing "penalty function" in the support n_k. The hyperparameter α interpolates between score-driven (i.e., BoN) and frequency-driven (i.e., SC) selection. When $\alpha = 0$, the penalty term vanishes and we select the answer group with the highest *mean* verification score; this differs from true BoN, which would pick the single response with the highest verification score. In the opposite extreme $\alpha \to \infty$, the penalty term dominates and the selection reduces to SC, i.e., choosing the answer with the largest support n_k. Empirically, we find $\alpha = 1.0$ to be an appropriate choice.

Shi & Jin (2025) leverages a generative verifier, Heimdall, requiring a total of $N\,(1+M) = O(NM)$ long CoT generations per problem, where M is the number of times each candidate solution is verified, leading to prohibitively high inference costs as N or M is scaled. To address this, we replace the costly generative verifier with a lightweight discriminative verifier. After sampling N candidate responses, we score each with the verifier. The cost of discriminative verification is negligible relative to the cost of sampling N responses. As we demonstrate in Section 4.3, this approach outperforms generative verification across most realistic compute budgets.

Building on Heimdall's bandit-style penalty, we use a Hoeffding-inspired lower-confidence bound (Hoeffding, 1963; Auer et al., 2002), replacing their $O((n_k M)^{-1})$ decay with a gentler $O(n_k^{-1/2})$ term. This $\sqrt{\cdot}$ form penalizes low-support answers less and decays quickly as n_k grows, boosting overall accuracy. Concretely, we score each answer group by

$$\mathrm{Score}(a_k) \;=\; \bar{r}(a_k) \;-\; \alpha\sqrt{\frac{\ln(N)}{n_k+1}},$$

and select the answer with the highest score. We formalize our approach in Algorithm 1.

Algorithm 1 Discriminative Pessimistic Verification (DPV@N)

Require: problem Q, solver LM, slate size N, verifier V, penalty weight α

Candidates $\leftarrow \{s_i\}_{i=1}^{N} \sim \mathrm{LM}(Q)$ ▷ **Stage 1: Generate Candidates**

Verifications $\leftarrow \{\, r_i = V(s_i)\}_{i=1}^{N}$ ▷ **Stage 2: Verify Candidates**

Partition $\{1,\ldots,N\}$ into clusters $\{\mathcal{C}_k\}$ by final answer a_k ▷ **Stage 3: Group Answers**

for each cluster $\mathcal{C}_k$ **do**

  $n_k \leftarrow |\mathcal{C}_k|$

  $\bar{r}(a_k) \leftarrow (1/n_k)\sum_{i\in\mathcal{C}_k} r_i$

  $\psi_k \leftarrow \Psi(N, n_k)$ ▷ e.g., $\Psi = \sqrt{\ln(N)/(n_k+1)}$

$k^* \leftarrow \arg\max_k \,[\,\bar{r}(a_k) - \alpha\,\psi_k\,]$ ▷ **Stage 4: Select Best Answer**

return a_{k^*}

4 Experiments

We test the performance of DPV on several challenging benchmarks: AIME2024, AIME2025, LiveBench Math (White et al., 2025), and GPQA (Rein et al., 2023). For each AIME problem, we sample 128 candidate responses no longer than 16k tokens from DeepSeek-R1-Distill-Qwen-32B. On LiveBench Math and GPQA, we sample only 64 candidate responses. We verify each response once with a discriminative verifier. During verification, we include the thinking content (i.e., the tokens between the <think> and </think> tags) during both training and inference; early experimentation motivated this decision. To ensure our metric estimates (e.g., Pass@N or DPV@N) are precise, we report the mean over 1000 resampled draws of size N per problem and give 95% confidence intervals computed with the binomial normal-approximation, treating every resampled draw as an independent Bernoulli trial. Our results are provided in Table 1.

Method	AIME2024	AIME2025	LiveBench Math	GPQA
Pass@1	67.0 ± 0.5	52.0 ± 0.6	62.1 ± 0.2	56.9 ± 0.2
SC@32	83.4 ± 0.4	66.6 ± 0.5	67.0 ± 0.2	63.5 ± 0.2
DV@32	79.3 ± 0.5	62.7 ± 0.5	67.4 ± 0.2	65.1 ± 0.2
DPV@32	$\mathbf{86.5 \pm 0.4}$	$\mathbf{70.2 \pm 0.5}$	$\mathbf{68.0 \pm 0.2}$	$\mathbf{66.2 \pm 0.2}$

Table 1: Accuracy rates of DeepSeek-R1-Distill-Qwen-32B with discriminative pessimistic verification ($N = 32$), compared to other inference methods. DPV and DV share the same underlying verifier, but for DPV we aggregate by final answer and subtract the pessimism penalty.

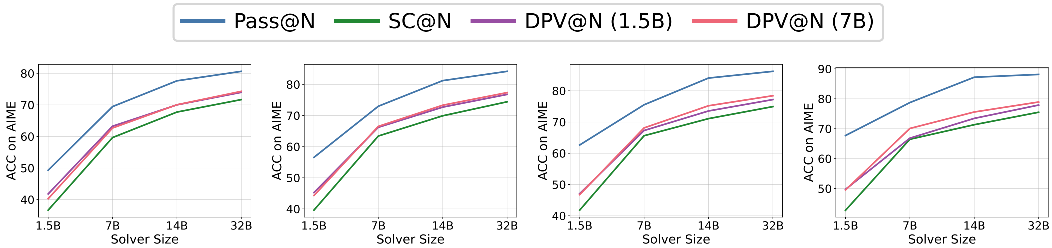
Across the board in Table 1, DPV outperforms competing selection methods under near-equivalent compute budgets. For example, on AIME2025, DPV@32 improves over Pass@1 by 18.2%, and beats SC@32 and DV@32 by 3.6% and 7.5%, respectively. Amazingly, even on an out-of-distribution task like GPQA, which includes questions on biology, physics, and chemistry, DPV@32 can outperform SC@32 by 2.7%.

In the following sections, we analyze the scaling properties of DPV. In Section 4.1, we study the impact of scaling the sizes of the solver and verifier models, while in Section 4.2, the focus is inference-time scaling properties (i.e., the number of candidate responses N and the solver's reasoning budget). Finally, in Section 4.3, we investigate the compute cost of DPV relative to comparison methods.

4.1 Scaling model sizes for DPV

To study the role of scaling the size of the solver model, we generate 128 candidate solutions per question in AIME2024 and AIME2025 using DeepSeek-R1-Distill-Qwen models with 1.5B, 7B, 14B, and 32B parameters. To isolate the effect of scaling the discriminative verifier, we train a second verifier initialized from DeepSeek-R1-Distill-Qwen-1.5B, and verify each candidate solution with both the 1.5B and 7B verifiers. We plot the results on AIME in Figure 4 for several values of N.

We observe that increasing the solver's size produces consistent but diminishing performance increases on AIME. Specifically, DPV and SC scale near-identically as the size of the solver is increased, with



(a) $N = 8$ (b) $N = 16$ (c) $N = 32$ (d) $N = 64$

Figure 4: Accuracy rates of DPV on AIME across various solver sizes and verifier sizes for several values of N. Pass@N and SC@N are included for comparison.

DPV maintaining a consistent edge over SC regardless of the solver or verifier's size, across various Ns. Moreover, we find that the performance difference between DPV with the 1.5B and 7B verifiers depends on N: when N is small, both verifiers perform similarly, but as N increases, the 7B verifier begins to outperform the 1.5B verifier. We believe this is because the larger verifier is more resistant to the long tail of persuasive but incorrect solutions that grows with N. Interestingly, we find that the 1.5B verifier consistently outperforms the 7B verifier when DeepSeek-R1-Distill-Qwen-1.5B is used as the solver model, likely because its responses are more in-distribution for the verifier.

4.2 Inference-time scaling of DPV

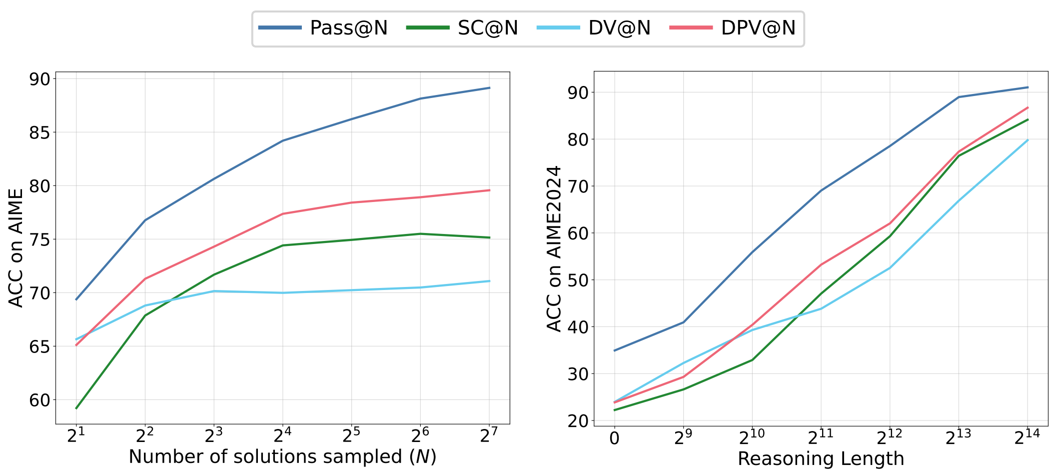


Figure 5: **Left:** Increasing the number of candidate results (N) sampled from DeepSeek-R1-Distill-Qwen-32B produces consistent but diminishing improvements on AIME 2024/2025. **Right:** The performance of DeepSeek-R1-Distill-Qwen-32B on AIME2024 scales logarithmically with the reasoning budget regardless of selection method. Here, $N = 32$.

We study whether DPV benefits from increased inference-time compute along two axes: the number of candidate solutions sampled from the solver and the reasoning budget allocated to the solver. First, we observe that scaling N produces consistent but diminishing improvements in performance on AIME (i.e., Pass@N increases). DV alone struggles to benefit from scaling N, with performance quickly saturating. On the other hand, DPV shows consistent improvements as more solutions are sampled, maintaining a 2.6% to 5.9% edge over SC as N is scaled from 2 to 128.

To control the reasoning budget, we use budget forcing (Muennighoff et al., 2025) and truncate the candidate solutions $T \in \{0, 512, 1024, 2048, 4096, 8192, 16384\}$ tokens after the opening think tag, manually append the closing think tag, then allow the model to continue generating. In doing so, we collect solutions under constrained reasoning budgets. We observe that even as the reasoning budget is scaled from 0 to 16k tokens, DPV maintains an edge over SC, even while DV falls off, showcasing the reliability of our method under various constraints.

4.3 The compute cost of DPV

We measure latency as a proxy for compute cost. Though prior work has simply focused on FLOPs (Singhi et al., 2025), natural language generation is often memory- and I/O-bound, which is not reflected in the FLOP count. Moreover, providers price GPUs by usage time, not FLOPs, so latency is the most direct proxy for real compute cost. Specifically, we calculate the average time taken to *generate and verify* candidate solutions on a single NVIDIA H100 NVL GPU based on the average input and output lengths of AIME2025. We leverage vLLM (Kwon et al., 2023) and its many optimizations, including dynamic batching, to reflect real-world usage.

	$N = 2$	$N = 4$	$N = 8$	$N = 16$	$N = 32$	$N = 64$	$N = 128$
Repeated Sampling	333.7	373.3	396.7	640.8	1103.0	2209.8	4218.6
DPV (or DV)	1.0	2.0	4.1	8.1	16.2	32.5	64.9
GPV (M=2)	402.2	578.7	1059.7	1990.9	3797.5	7715.4	15260.0

Table 2: The average wall-clock time for repeatedly sampling N candidate solutions, as well as the average time to verify each candidate solution using DPV (or DV) and GPV. For GPV, two verifications are sampled per candidate solution.

Table 2 showcases the average time to sample and verify N candidate solutions using various methods. Specifically, we consider SC, DPV and its non-pessimistic counterpart DV, as well as a pessimistic method for generative verification (GPV) proposed by Shi & Jin (2025). SC uses no additional compute beyond repeated sampling, while discriminative verification techniques use just slightly more, and generative verification uses significantly more. For example, sampling $N = 8$ candidate responses to a question from AIME2025 takes 396.7s, on average. Meanwhile, verifying those 8 solutions with DV or DPV takes just 4.1s, but verifying the same 8 solutions with GPV takes 1059.7s, a 258-fold increase.

Figure 1 displays the performance of these methods on AIME2025 under *equalized compute budgets*. We observe that DPV consistently outperforms SC by between 3.4% and 5.2% for a negligible amount (<1.5%) of additional compute. DV uses the same amount of compute as DPV, but its performance quickly saturates as more candidate solutions are sampled. Importantly, under practical compute limitations (e.g., <20 minutes per problem), DPV actually outperforms GPV by as much as 10%. This is because DPV allocates nearly all of the budget towards sampling candidate solutions, while GPV splits its compute budget between sampling and verifying candidates. Under modest compute budgets, scaling the number of candidate solutions produces greater returns than scaling verifications; even an oracle-level verifier will fail to produce the correct answer if no correct solutions were sampled. With a large enough budget, however, the gain from sampling additional candidates begins to saturate, and GPV begins to dominate DPV.

5 Related Work

LLM Verifiers LLM-based verifiers can be broadly categorized into generative and discriminative approaches. Generative verifiers use large language models as judges that assess the correctness or quality of outputs by generating natural language rationales. A growing body of work explores this direction, employing LLMs as judges for modeling human preferences (Dubois et al., 2024; Zheng et al., 2024; Li et al., 2024; Wang et al., 2023c; Kim et al., 2023, 2024; Li et al., 2023; Zhu et al., 2023b; Mahan et al., 2024), or as verifiers for evaluating solution correctness in reasoning tasks (Zhang et al., 2024c; Singhi et al., 2025; Shi & Jin, 2025; Saha et al., 2025).

In contrast, discriminative verifiers—such as reward models—assign scalar scores to candidate responses based on human preference data (Christiano et al., 2017; Ziegler et al., 2019; Zhu et al., 2023a; Liu & Zeng, 2024; Wang et al., 2024; Park et al., 2024; Han et al., 2024). These models are central to reinforcement learning from human feedback and are also used to rank or select responses in BoN inference settings (Lightman et al., 2023; Wang et al., 2023a; Luo et al., 2024; Saunders et al., 2022; Uesato et al., 2022; Yu et al., 2024). Together, generative and discriminative verifiers provide complementary paradigms for evaluating, selecting, and aligning LLM outputs at inference time.

LLM Reasoning A substantial body of work has investigated improving the mathematical reasoning capabilities of LLMs through training Cobbe et al. (2021); Guan et al. (2025); Hosseini et al. (2024); Lightman et al. (2023); Pang et al. (2024); Ye et al. (2025); Luo et al. (2025b,a), test-time scaling Snell et al. (2024); Brown et al. (2024); Setlur et al. (2024), or a combination of both Zhang et al. (2024b); Guan et al. (2025); Xie et al. (2024); Zhang et al. (2024a). Following the release of o1 OpenAI (2024), there has been a surge of interest in test-time scaling methods for LLM reasoning Snell et al. (2024); Brown et al. (2024); Singhi et al. (2025); Zhao et al. (2025), which improve performance by sampling multiple solutions and aggregating them via majority voting or LLM-based verification. Our work builds on this line of research, demonstrating that discriminative LLM verifiers can serve as an effective and efficient verification approach for test-time scaling in complex math reasoning tasks.

6 Conclusion

In this work, we demonstrated that a weak discriminative verifier paired with a pessimistic penalty term to discount answers with low support can enable strong test-time scaling. For example, on AIME2025 our method beats self-consistency by between 3.4% and 5.2% for a negligible amount (<1.5%) of additional compute, and even outperforms generative verification techniques by up to 10% under practical compute limits. These results show that carefully-designed discriminative verification offers an immediately deployable, compute-efficient path to stronger LLM reasoning.

Limitations and Broader Impacts

Limitations Our method improves answer selection only when at least one correct candidate is present, so its ceiling is still bounded by the solver's Pass@N. Additionally, like SC, our method assumes that responses can be clustered into equivalence classes and thus would likely not be suitable for domains lacking a reliable mechanism for determining answer equivalence (e.g., open-ended natural-language tasks). Also, under extreme compute budgets, generative verification techniques outperform our proposed approach. Lastly, our compute analysis between discriminative and generative verification is grounded in current software and hardware; with rapidly advancing progress on both fronts, generative verification is sure to grow more efficient.

Broader Impacts Our proposed method enables highly efficient yet effective test-time scaling. This may lower the hardware barrier for academic labs or other groups that need strong reasoning, but cannot afford massive inference clusters. On the flip side, better low-cost reasoning may accelerate misuse scenarios, which can be mitigated by techniques such as rate-limiting, watermarking, or alignment training.

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A Additional Technical Details

Our training data is based on a subset of Numina-Math (LI et al., 2024), which was released under an Apache license 2.0. DeepSeek-R1 responses were collected from Mattern et al. (2025) (also Apache 2.0). Meanwhile, the majority of the responses from six DeepSeek-R1-Distill models, DeepScaleR-1.5B-Preview, and the two QwQ models were generated on a local cluster of NVIDIA A100 GPUs, with a minority coming from 3rd party API providers.

Our evaluation datasets are AIME2024 (MIT), AIME2025 (MIT), LiveBench-Math (White et al., 2024) (Apache 2.0), and GPQA (Rein et al., 2023) (CC-by-4.0). Combined, they include 596 questions. We decontaminate the training dataset by excluding any problem whose fuzzy-match similarity to an entry in our evaluation sets exceeds 80. For each AIME problem, we sample 128 candidate solutions, while on LiveBench Math and GPQA, we sample only 64 candidate solutions.

When rolling out solutions during training and evaluation, we follow the model's usage recommendations, namely prefilling the opening think token, sampling with a temperature of 0.6 and a top-p value of 0.95, and instructing the model to output its final answer within `\boxed{}`.

Our 1.5B and 7B discriminative verifiers were trained on 4xA100s and 4xH200s, respectively. For both, we use the hyperparameters listed in Table 3.

Hyper-parameter	**Value**
Global batch size	16
Gradient accumulation steps	4
LR	5×10^{-5}
LR scheduler	Linear with 20 warmup steps
Optimizer (AdamW)	$\beta_1 = 0.9,\ \beta_2 = 0.999$
λ	0.01
Max gradient norm	1.0

Table 3: Hyper-parameters for training discriminative verifiers.

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